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COMPARISON OF THE TECHNIQUES USED IN THE NEWMARK ANALYSIS OF NONLINEAR STRUCTURES

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ABSTRACT

This paper compares the techniques used to minimize the error due to the unbalanced forces that appear in the Newmark analysis of nonlinear structures. First, a brief review of the Newmark's method and the error minimization techniques used in the Newmark analysis are given. The methods compared are the Newton-Raphson iteration, pseudo-force method and the unbalanced force correction method. Then, three SDOF and one three-DOF simplified bridge model, all consist of various nonlinear elements, are analyzed using El Centro earthquake ground acceleration data. The performances of each method are graphically compared for several time steps by defining a cumulative error index based on benchmark analyses, which are carried out with a very small time step. The nonlinear models considered herein are a bilinear element, a triliniear element, known as original Takeda model and a very sophisticated high damping rubber bearing element, which is governed by several differential equations. Based on the numerical simulations, cumulative error performances and time-effectiveness of each method are discussed. It is found that each method has its own merits and disadvantages mostly determined by the type of nonlinearity.

Keywords: Newmark's method, nonlinear analysis, pseudo-force, Newton-Raphson iteration, unbalanced force correction

INTRODUCTION

Newmark's method (Newmark, 1959) converts differential equations of motion of a structure to a simpler form, which is solved algebraically and incrementally. Response quantities obtained solving this algebraic equation possess error for nonlinear systems since the nonlinear element forces used in this equation are different from those obtained using the nonlinear element models. The differences between these two forces are known as *unbalanced forces*. Generally, additional methods are employed to minimize either the error or the unbalanced forces. The most well-known method among them is Newton-Raphson iteration. However, Newton-Raphson iteration cannot be employed for systems with special devices or members, which cannot be modeled with a constant damping and a nonlinear force-displacement relation. For these systems, other methods such as pseudo-force method or unbalanced force correction method are employed. For systems with different type of nonlinearities, hybrid techniques can also be used. There are

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several examples of the applications (Stricklin *et al.*, 1971; Stricklin & Haisler, 1977; Nelson and Mak 1982; Molnar et al. 1976; Ohtori and Spencer, 1999; Nagarajaiah *et al.*, 1991) and state-of-the-art papers (*e.g.* Subbaraj and Dokainish, 1989) of these methods in literature. It will be also useful to understand the performance characteristics and time efficiencies of these methods for different type of nonlinearities those frequently utilized in the civil engineering practice.

This paper compares the techniques used to minimize the errors due to the unbalanced forces in Newmark's method analyzing three SDOF systems and one three-DOF bridge model formed with three types of nonlinear elements. First, a review of the Newmark's method and the methods used to minimize the error are given. These methods are Newton-Raphson iteration, pseudo-force method and unbalanced force correction method. Then, three SDOF systems and a three-DOF bridge model are defined. The nonlinear elements used in these systems are a bilinear model, a recently developed high damping rubber bearing model (Abé *et al.* 2004) and original a triliniear Takeda model (Takeda *et al*. 1970). Each system is analyzed by the aforementioned techniques for several time intervals using El Centro earthquake ground acceleration. An error index is defined based on the RMS error of the responses and forces, and benchmark analyses for each system. Finally, performances and time efficiencies of these methods are discussed for the systems considered. MATLAB (MATLAB 2001) is used as the programming language.

A REVIEW OF THE ANALYSIS METHODS

In this section, Newmark's method and the error minimization techniques are summarized. Equation of motion of most of the structural systems is in the form of

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{F}_\mathrm{s}=\mathbf{P}(t) \tag{1}$$

where $\mathbf{x}$ is the displacement vector, $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{F}_\mathrm{s}$ is the internal force vector, which is a function of the displacement, velocity and acceleration, $\mathbf{P}$ is the external force vector, and the dot designates the time derivative. Subtracting the equation of motion at time t from the equation of motion at time $t+\Delta t$, one obtains the incremental formulation of the equation of motion as follows:

$$\mathbf{M}\Delta\ddot{\mathbf{x}}^t+\mathbf{C}\Delta\dot{\mathbf{x}}^t+\Delta\mathbf{F}_\mathrm{s}^t=\Delta\mathbf{P}^t \tag{2}$$

where $\Delta\mathbf{x}^t=\mathbf{x}^{t+1}-\mathbf{x}^t$, $\Delta\dot{\mathbf{x}}^t=\dot{\mathbf{x}}^{t+1}-\dot{\mathbf{x}}^t$, $\Delta\ddot{\mathbf{x}}^t=\ddot{\mathbf{x}}^{t+1}-\ddot{\mathbf{x}}^t$, $\Delta\mathbf{F}_\mathrm{s}^t=\mathbf{F}_\mathrm{s}^{t+1}-\mathbf{F}_\mathrm{s}^t$ and $\Delta\mathbf{P}^t=\mathbf{P}^{t+1}-\mathbf{P}^t$. Newmark's method uses the following assumptions:

$$\dot{\mathbf{x}}^{t+1}=\dot{\mathbf{x}}^t+[(1-\gamma)\Delta t]\ddot{\mathbf{x}}^t+(\gamma\Delta t)\ddot{\mathbf{x}}^{t+1} \quad \text{and} \quad \mathbf{x}^{t+1}=\mathbf{x}^t+\Delta t\dot{\mathbf{x}}^t+[(0.5-\beta)(\Delta t)^2]\ddot{\mathbf{x}}^t+\beta(\Delta t)^2\ddot{\mathbf{x}}^{t+1} \tag{3}$$

where the parameters β and γ define the variation of the acceleration over the time interval Δt. Incremental velocity and acceleration can readily be obtained as

$$\Delta\dot{\mathbf{x}}^t=\frac{\gamma}{\beta\Delta t}\Delta\mathbf{x}^t-\frac{\gamma}{\beta}\dot{\mathbf{x}}^t+\Delta t\left(1-\frac{\gamma}{2\beta}\right)\ddot{\mathbf{x}}^t \quad \text{and} \quad \Delta\ddot{\mathbf{x}}^t=\frac{1}{\beta\Delta t^2}\Delta\mathbf{x}^t-\frac{1}{\beta\Delta t}\dot{\mathbf{x}}^t-\frac{1}{2\beta}\ddot{\mathbf{x}}^t \tag{4}$$

Substituting Eq. (4) into Eq. (2), one obtains the algebraic form of the equation of motion as

$$\mathbf{A}\Delta\mathbf{x}^t+\Delta\mathbf{F}_\mathrm{s}^t=\Delta\hat{\mathbf{P}}^t \qquad \text{where} \tag{5}$$

$$\mathbf{A}=\frac{\gamma}{\beta\Delta t}\mathbf{C}+\frac{1}{\beta\Delta t^2}\mathbf{M} \quad \text{and} \quad \Delta\hat{\mathbf{P}}^t=\Delta\mathbf{P}^t+\left(\frac{1}{\beta\Delta t}\mathbf{M}+\frac{\gamma}{\beta}\mathbf{C}\right)\dot{\mathbf{x}}^t+\left[\frac{1}{2\beta}\mathbf{M}+\Delta t\left(\frac{\gamma}{2\beta}-1\right)\mathbf{C}\right]\ddot{\mathbf{x}}^t. \tag{6}$$

Eq. (5) is algebraic but nonlinear since it can only be solved for $\Delta\mathbf{x}^t$ if $\Delta\mathbf{F}_s^t$ — a function of $\Delta\mathbf{x}^t$, $\Delta\dot{\mathbf{x}}^t$ and $\Delta\ddot{\mathbf{x}}^t$ — is known. Therefore, Eq. (5) is solved approximately in two steps:

Step 1: An assumption is made for $\Delta\mathbf{F}_s^t$, and $\Delta\mathbf{x}^t$ is solved. The difference between the assumed $\Delta\mathbf{F}_s^t$ and the actual $\Delta\mathbf{F}_s^t$ is called *unbalanced force*. Assumptions can be made based on tangential stiffness or time derivative of internal force, or it can be $\Delta\mathbf{F}_s^t$ obtained in the previous time step, *i.e.*, $\Delta\mathbf{F}_s^{t-1}$.

Step 2: Additional techniques are utilized to minimize the unbalanced forces or the resulting error. There are two types of techniques utilized; iterative techniques such as Newton-Raphson iteration and pseudo-force method and single-step techniques such as unbalanced force correction method.

Here, the abbreviations used throughout the review of these methods are given. $\Delta\mathbf{F}_s^{t,i,\mathrm{aif}}$: Assumed incremental internal force at iteration *i*. Here, *aif* stands for *assumed internal force*. If an iterative method is not used, *i* can be omitted. $\Delta\mathbf{x}^{t,i}$: Incremental displacement obtained from equation (5) using $\Delta\mathbf{F}_s^{t,i,\mathrm{aif}}$. $\Delta\mathbf{F}_s^{t,i,\mathrm{eif}}$: Incremental internal force vector obtained from element models using $\Delta\mathbf{x}^{t,i}$, $\Delta\dot{\mathbf{x}}^{t,i}$ and $\Delta\ddot{\mathbf{x}}^{t,i}$. Here, *eif* stands for *estimated internal force*. $\Delta\mathbf{F}_s^{t,i,\mathrm{euf}}$: Estimated unbalanced force; the difference between $\Delta\mathbf{F}_s^{t,i,\mathrm{aif}}$ and $\Delta\mathbf{F}_s^{t,i,\mathrm{eif}}$. $\Delta\mathbf{F}_s^{t,i,\mathrm{auf}}$: Actual unbalanced force; the difference between $\Delta\mathbf{F}_s^{t,i,\mathrm{aif}}$ and $\Delta\mathbf{F}_s^t$.

Assumptions Used in the First Step

Assumption Based on the Tangential Stiffness (*TS*)

Assume that

$$\Delta\mathbf{F}_s^{t,1,\mathrm{aif}} = \mathbf{K}_T^{t,1}\Delta\mathbf{x}^{t,1} \tag{7}$$

where $\mathbf{K}_T^{t,1}$ is the tangential stiffness matrix computed at the beginning of the time interval. Substituting Eq. (7) into Eq. (5), one obtains the incremental displacement and the estimated unbalanced force as

$$\Delta\mathbf{x}^{t,1} = \Delta\hat{\mathbf{P}}^t(\mathbf{A}+\mathbf{K}_T^{t,1})^{-1} \quad \text{and} \quad \mathbf{F}_s^{t,1,\mathrm{euf}} = \Delta\mathbf{F}_s^{t,1,\mathrm{aif}} - \Delta\mathbf{F}_s^{t,1,\mathrm{eif}} = \mathbf{K}_T^{t,1}\Delta\mathbf{x}^{t,1} - \Delta\mathbf{F}_s^{t,1,\mathrm{eif}}\,. \tag{8}$$

Assumption Based on the Internal Force Obtained in the Previous Time Step (*IFPTS*)

Internal force is assumed to be equal to the internal force obtained in the previous time step:

$$\Delta\mathbf{F}_s^{t,1,\mathrm{aif}} = \Delta\mathbf{F}_s^{t-1} \tag{9}$$

The incremental displacement and the estimated unbalanced force become

$$\Delta\mathbf{x}^{t,1} = (\Delta\hat{\mathbf{P}}^t - \Delta\mathbf{F}_s^{t-1})\mathbf{A}^{-1} \quad \text{and} \quad \mathbf{F}_s^{t,1,\mathrm{euf}} = \Delta\mathbf{F}_s^{t-1} - \Delta\mathbf{F}_s^{t,1,\mathrm{eif}}\,. \tag{10}$$

Assumption Based on the Time Derivative (*TD*) *of the Internal Force Vector.*

Assume that

$$\Delta\mathbf{F}_s^{t,1,\mathrm{aif}} = \frac{\partial\mathbf{F}_s^t}{\partial t}\Delta t \tag{11}$$

The incremental displacement and the estimated unbalanced force become

$$\Delta\mathbf{x}^{t,1} = \left(\Delta\hat{\mathbf{P}}^t - \frac{\partial\mathbf{F}_s^t}{\partial t}\Delta t\right)\mathbf{A}^{-1} \quad \text{and} \quad \mathbf{F}_s^{t,1,\mathrm{euf}} = \frac{\partial\mathbf{F}_s^t}{\partial t}\Delta t - \Delta\mathbf{F}_s^{t,1,\mathrm{eif}} \tag{12}$$

Error Minimization Techniques Used in the Second Step

Newton-Raphson (*NR*) *Iteration*

Unbalanced force $\mathbf{F}_{\mathrm{s}}^{t,n-1,\mathrm{euf}}$ is considered as an external force as

$$\mathbf{A}\Delta\mathbf{x}^{t,n} + \Delta\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{aif}} = \mathbf{F}_{\mathrm{s}}^{t,n-1,\mathrm{euf}} \tag{13}$$

where $\Delta\mathbf{x}^{t,n}$, $\Delta\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{aif}}$ are the displacement and internal force vectors due to $\mathbf{F}_{\mathrm{s}}^{t,n-1,\mathrm{euf}}$. Note that in the first step $n=1$ is used (see Eqs. (7) to (12)). Eq. (13) is solved assuming the internal force vector is based on the tangential stiffness as

$$\Delta\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{aif}} = \mathbf{K}_{T}^{t,1}\Delta\mathbf{x}^{t,n}. \tag{14}$$

The displacement due to $\mathbf{F}_{\mathrm{s}}^{t,n-1,\mathrm{euf}}$ becomes

$$\Delta\mathbf{x}^{t,n} = \mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{euf}}(\mathbf{A} + \mathbf{K}_{T}^{t,1})^{-1}. \tag{15}$$

$\Delta\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{eif}}$ is estimated using $\Delta\mathbf{x}^{t,n}$, $\Delta\dot{\mathbf{x}}^{t,n}$ and $\Delta\ddot{\mathbf{x}}^{t,n}$. The unbalanced force for iteration $n \geq 2$ is estimated as

$$\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{euf}} = \Delta\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{aif}} - \Delta\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{eif}} \quad \textit{or} \quad \mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{euf}} = \mathbf{K}_{T}^{t,1}\Delta\mathbf{x}^{t,n} - \Delta\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{eif}}. \tag{16}$$

This iteration is carried out until an error criterion based on $\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{euf}}$ or $\Delta\mathbf{x}^{t,n}$ is satisfied. Let N be the total iteration number. Final incremental displacement and incremental internal force vectors are

$$\Delta\mathbf{x}^{t} = \sum_{j=1}^{N}\Delta\mathbf{x}^{t,j} \quad \textit{and} \quad \Delta\mathbf{F}_{\mathrm{s}}^{t} = \sum_{j=1}^{N}\Delta\mathbf{F}_{\mathrm{s}}^{t,j,\mathrm{eif}} \tag{17}$$

If $\mathbf{K}_{T}^{t,n}$ is used during iteration n instead of $\mathbf{K}_{T}^{t,1}$, this method is called *modified Newton-Raphson iteration*.

Pseudo-force (*PF*) *Method*

The internal force is assumed to be the estimated internal force obtained in the previous iteration. Note that for $n = 2$, internal force obtained in the first step is used. The displacement is computed as,

$$\Delta\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{aif}} = \Delta\mathbf{F}_{\mathrm{s}}^{t,n-1,\mathrm{eif}} \quad \text{and} \quad \Delta\mathbf{x}^{t,n} = (\Delta\hat{\mathbf{P}}^{t} - \Delta\mathbf{F}_{\mathrm{s}}^{t,n-1,\mathrm{eif}})\mathbf{A}^{-1}. \tag{18}$$

$\Delta\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{eif}}$ is computed using $\Delta\mathbf{x}^{t,n}$, $\Delta\dot{\mathbf{x}}^{t,n}$ and $\Delta\ddot{\mathbf{x}}^{t,n}$. This iteration is carried out until an error criterion based on $\mathbf{F}_{\mathrm{s}}^{t,n,\mathrm{euf}}$ or $\Delta\mathbf{x}^{t,n}$ is satisfied. Let N be the total number of iterations. The final incremental displacement and the internal force are $\Delta\mathbf{x}^{t} = \Delta\mathbf{x}^{t,N}$ and $\Delta\mathbf{F}_{\mathrm{s}}^{t} = \Delta\mathbf{F}_{\mathrm{s}}^{t,N,\mathrm{eif}}$.

Unbalanced Force Correction (*UFC*) *Method*

This method is not an iterative method. The estimated unbalanced force obtained in the first step at time *t*, is applied to the system in the next time step, and an updated $\Delta\hat{\mathbf{P}}^{t+1}$ given by $\Delta\tilde{\mathbf{P}}^{t+1} = \Delta\hat{\mathbf{P}}^{t+1} + \mathbf{F}_{\mathrm{s}}^{t,1,\mathrm{euf}}$ is used instead of the original $\Delta\hat{\mathbf{P}}^{t+1}$. Therefore,

$$\Delta\mathbf{x}^{t+1} = (\Delta\tilde{\mathbf{P}}^{t+1} - \Delta\mathbf{F}_{\mathrm{s}}^{t+1,\mathrm{aif}})\mathbf{A}^{-1}. \tag{19}$$

Here, $\Delta\mathbf{F}_{\mathrm{s}}^{t+1,\mathrm{aif}}$ can be one of the assumptions given in the first step. Also, note that iteration index *i* is omitted in this formulation.

Practical Applications

In general, a structural system may consists of several types of nonlinear elements. In this case, an hybrid method can be used for analysis. To illustrate this concept, consider the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{F}_{s,1} + \mathbf{F}_{s,2} = \mathbf{P}(t) \tag{20}$$

where $\mathbf{F}_{s,1}$ and $\mathbf{F}_{s,2}(\mathbf{x})$ represent internal forces of two groups of elements. Let

$$\mathbf{F}_{s,1} = \mathbf{C}_1\dot{\mathbf{x}} + \mathbf{F}^x_{s,1}(\mathbf{x}) \quad \text{and} \quad \frac{d\mathbf{F}^x_{s,1}}{d\mathbf{x}} \text{ is defined.} \tag{21}$$

Here, $\mathbf{C}_1$ is a constant damping matrix, and $\mathbf{F}^x_{s,1}$ is a displacement dependent function. Let $\mathbf{F}_{s,2}$ be a function that cannot be expressed in terms of a constant damping matrix and a displacement-dependent function as in Eq. (21). Moving $\mathbf{F}_{s,2}(\mathbf{x})$ to the right hand side of equation of motion and using incremental formulations, one obtains

$$\mathbf{A}\Delta\mathbf{x}^t + \mathbf{F}^x_{s,1}(\mathbf{x}) = \Delta\bar{\mathbf{P}}^t \quad \text{where} \quad \Delta\bar{\mathbf{P}}^t = \Delta\hat{\mathbf{P}}^t - \mathbf{F}_{s,2}(\mathbf{x}). \tag{22}$$

and $\mathbf{C}_1$ is included in matrix $\mathbf{A}$. Noting that the first equation in (22) is in the form of Eq. (5), one can utilize NR iteration over $\mathbf{F}^x_{s,1}(\mathbf{x})$, with an initial assumption based on the tangential stiffness, for a given $\Delta\bar{\mathbf{P}}^t$. For $\Delta\hat{\mathbf{P}}^t$, one can assume a $\mathbf{F}_{s,2}(\mathbf{x})$ based on its value in the previous time step and carry a PF iteration over $\mathbf{F}_{s,2}(\mathbf{x})$. Clearly, the final method is a hybrid technique that includes two nested iterations, where the outer is the PF iteration, and the inner is the NR iteration.

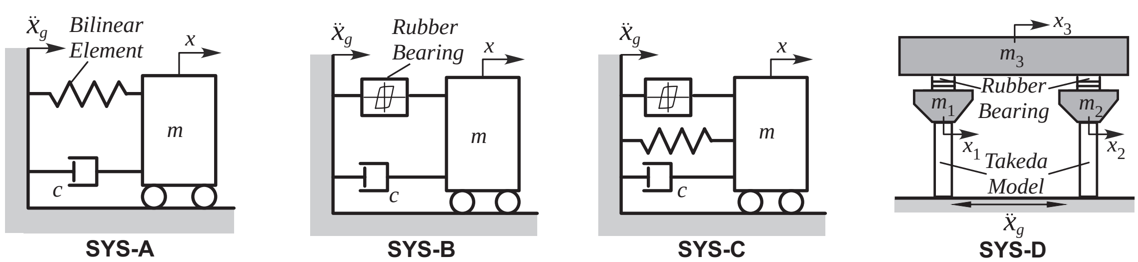

FIG 1. Schematic representation of the systems analyzed

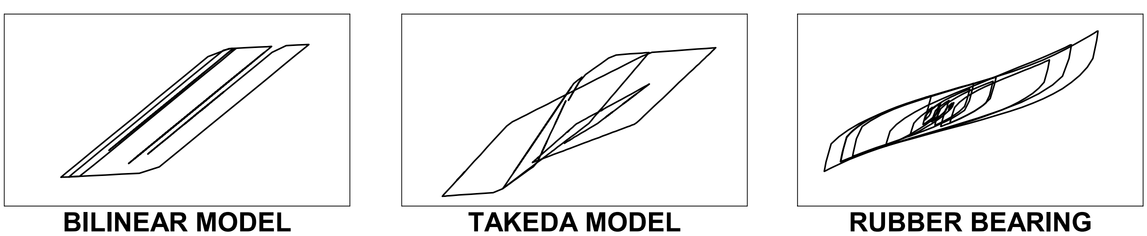

FIG. 2. Representative behaviors of the models used in the systems

NUMERICAL EXAMPLES

In this section, three-SDOF and one three-DOF system (Fig. 1) are analyzed using the methods described above. The structural parameters for these systems are given in Table 1. The rubber bearing element model is evaluated with a fourth order Runge-Kutta solver. Representative behaviors of the nonlinear elements are shown in Fig. 2. The methods used to analyze the systems are summarized in Table 2. For the PF and NR methods, only three iterations are carried out. The Newmark parameters are set to $\beta = 1/6$ and $\gamma = 1/2$. El Centro ground motion is used as the excitation. An error index is defined based on benchmark analyses. The benchmark analyses use NR iteration for SYS-A and NR and PF iterations for the other systems with $\Delta t = 0.0005$ sec. For each of the benchmark system, five iterations are carried out. The error index is defined as

$$I = \mathrm{RMS}\left\{ \mathrm{RMS}\left(\frac{x_i - x_i^{bm}}{x_i^{bm}}\right) \quad \mathrm{RMS}\left(\frac{\dot{x}_i - \dot{x}_i^{bm}}{\dot{x}_i^{bm}}\right) \quad \mathrm{RMS}\left(\frac{\ddot{x}_i - \ddot{x}_i^{bm}}{\ddot{x}_i^{bm}}\right) \quad \mathrm{RMS}\left(\frac{F_j - F_j^{bm}}{F_j^{bm}}\right) \right\} \tag{23}$$

where i corresponds to the DOF number and j corresponds to the nonlinear member number. Error index I, and the duration of the analysis for each system and method are plotted for several time intervals as shown in Fig. 3.

Table 1. Summary of the structural parameters of the systems analyzed

Mass	Damping and Stiffness		Rubber Bearing*
	Bilinear	**Takeda**	
m = 20 tons m_1 = 10 tons m_2 = 15 tons m_3 = 125 tons	k_1 = 3160 ton-f/m k_2 = 200 ton-f/m f_y=100 ton-f c = 10 ton-f·sec/m	k_1 = 3000 ton-f/m k_2 = 1000 ton-f/m k_3 = 100 ton-f/m x_y = 1 cm x_c = 3 cm c = 10 ton-f·sec/m	$Y = Y_0\left(1+\gamma\left(\frac{\lvert U\rvert}{U_0}\right)^p\right)$, $\frac{S}{Y} = \left(\alpha_0 - \beta\left(\frac{U_{\max}}{U_0}\right)^q\right)\left(\frac{U}{U_0} - \frac{F_1}{Y}\right)$ $\dot{F}_2 = \dot{U}k - \frac{k}{\eta}F_2$, $\dot{F}_1 = \frac{Y}{U_0}\left\{\dot{U} - \lvert\dot{U}\rvert\, sign\left(\frac{F_1 - S}{Y}\right)\left(abs\left(\frac{F_1 - S}{Y}\right)\right)^n\right\}$ $F = F_1 + F_2$ where Y_0 = 1.77 tons, U_0 = 5.50 mm, α = 0.778, β = 0.556, γ = 0.00102, k = 0.0197 ton/mm, n = 0.232, p = 2.71, q = 0.099, η = 4.8832 ton.sec/mm and $U_{\max} = \max(\lvert U\rvert)$.

* This is an early version of a bearing model developed by the Bridge and Structure Laboratory of Civil Engineering Department of The University of Tokyo for a bearing of size 21(W)×21(L)×18.2(H) cm under an axial load of 40 kg-f/cm^2. Contact to Prof. Masato Abe at masato@bridge.t.u-tokyo.ac.jp for further information.

Table 2. Summary of the methods compared

	Method ↓	Bilinear Element	
		1st Step	2nd Step
SYS-A	PF	IFPTS	PF
	UFC	IFPTS	UFC
	NR	TS	NR

	Method ↓	Rubber Bearing	
		1st Step	2nd Step
SYS-B	PF	IFPTS	PF
	UFC	IFPTS	UFC
	PF-TD	TD	PF
	UFC-TD	TD	UFC

	Method ↓	Bilinear Element		Rubber Bearing	
		1st Step	2nd Step	1st Step	2nd Step
SYS-C	NR-PF	TS	NR	IFPTS	PF
	NR-UFC	TS	NR	IFPTS	UFC
	PF	IFPTS	PF	IFPTS	PF
	UFC	IFPTS	UFC	IFPTS	UFC
	UFC-TD	TD	UFC	TD	UFC

	Method ↓	Takeda Element		Rubber Bearing	
		1st Step	2nd Step	1st Step	2nd Step
SYS-D	NR-PF	TS	NR	IFPTS	PF
	PF	IFPTS	PF	IFPTS	PF
	UFC	IFPTS	UFC	IFPTS	UFC

The followings are observed based on the results:

SYS-A : All the methods have same performance while UFC method is more time-efficient.

SYS-B : For $\Delta t > 0.005$ sec, PF iteration have better performance than UFC. Using TD in the 1st step improves UFC method.
SYS-C : For $\Delta t > 0.005$ sec, NR-PF and PF have best performance, and methods with UFC lowers the performance. Similar to SYS-B, using TD improves UFC.
SYS-D : For $\Delta t > 0.005$ sec, NR and PF methods have better performance.
All Systems: Methods with UFC has better time efficiency for all time steps. For $\Delta t < 0.005$ sec, all methods show same performance.

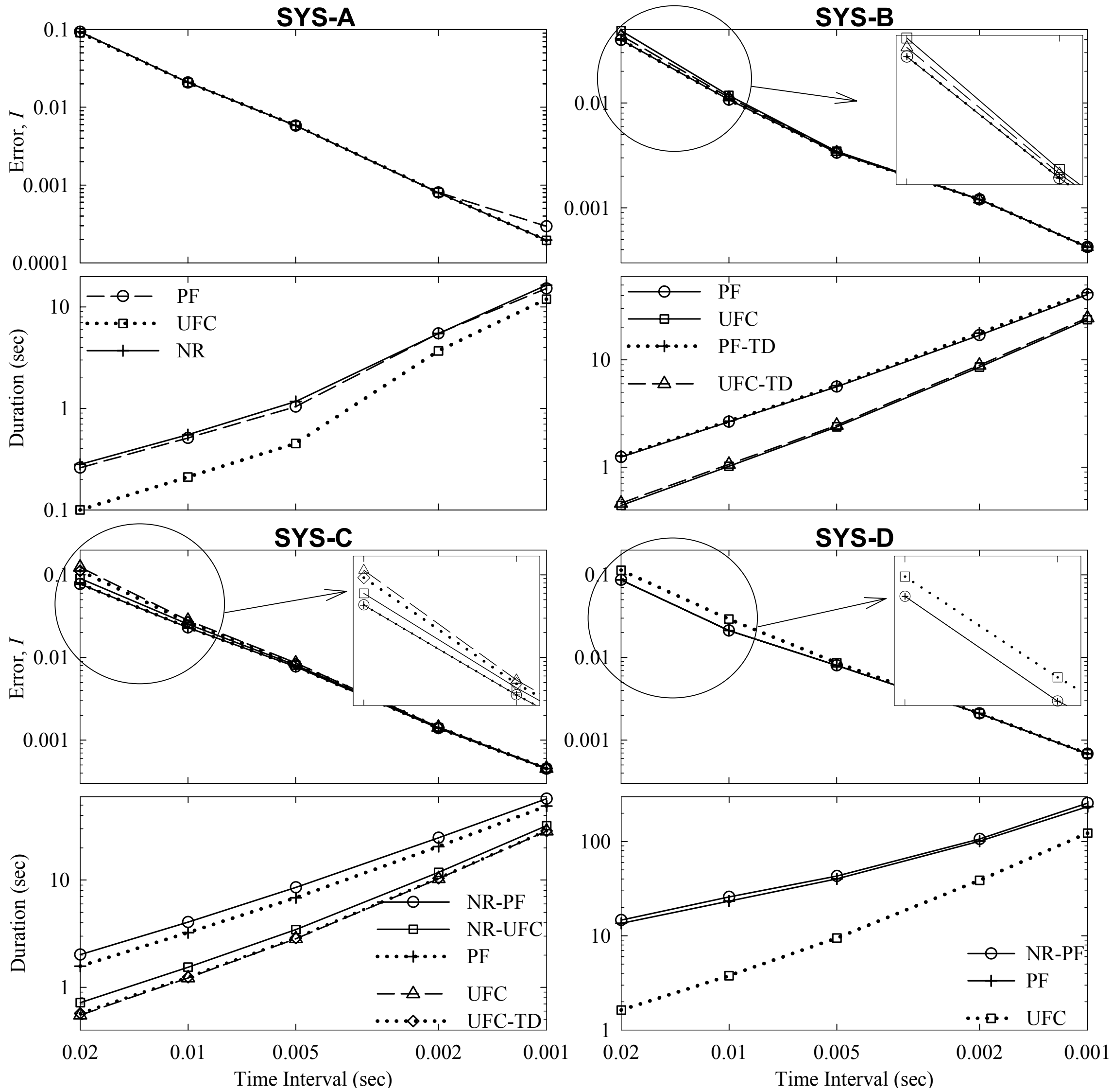

FIG. 3 Error of the responses with respect to the benchmark analysis and analysis durations for several time intervals.

The methods described above have their own advantages and disadvantages. For example, NR iteration can only be applied to element forces those are in the form of (21) (*e.g.*, piecewise linear element models). PF shows same error and time performance as NR for piecewise linear models, while it may not be time efficient for more complex models. For some problems where

MATLAB and Simulink (MATLAB, 2001) are used as software tools, it may be practical to employ UFC to avoid iterations. On the other hand, UFC method does not have a dynamic error checking scheme and may yield unpredictable results for very stiff (highly nonlinear) systems.

CONCLUSIONS

Several techniques used for Newmark's analysis of nonlinear structure are investigated for several type of nonlinearities. The accuracy and time-efficiency of the methods are compared for several time steps. It is found that, PF method is both a practical and efficient solution for several type of nonlinearities. UFC method show better time-efficiency and have error performance similar to other methods for small time steps. However, it should be used carefully since it does not have an error-checking algorithm.

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